Supplementary Information for "The fracture energy of ruptures driven by flash heating"

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1 Temperature asymptotics for the slip on a plane approximation

Large times. Introducing a nondimensional temperature rise $\theta = (T - T_0)/(T_\mathrm{w} - T_0)$ and a nondimensional time $u = t/t_\mathrm{w}^\mathrm{SP}$, Equation (9) of the main text is conveniently rewritten as

$$\theta(u) = \frac{1}{2\sqrt{\pi}} \int_0^u \frac{(1-\theta(u'))^2}{\sqrt{u-u'}} du'. \tag{1}$$

Unfortunately, we could not find an analytical solution to Equation (1) in the general case. However, we determined a useful asymptotic solution for large u by looking for solutions of the form

$$\theta(u) = 1 - \epsilon(u) \tag{2}$$

with $\epsilon(u) \to 0$ as $u \to \infty$. Equation (1) is then rewritten as

$$1 - \epsilon(u) = \frac{1}{2\sqrt{\pi}} \int_0^u \frac{\epsilon(u')^2}{\sqrt{u-u'}} du'. \tag{3}$$

This can be solved approximately by looking for simple expressions for $\epsilon(u)$ which ensure that leading order terms on each side are balanced correctly. In a first approximation, we should find $\epsilon(u)$ so that the integral on the l.h.s. is of $O(1)$. By inspecting the integral on the l.h.s., we can see that $\epsilon(u) \propto u^{-1/4}$ will indeed integrate to a constant. Matching the leading order terms leads to the following asymptotic approximation for $\epsilon(u)$:

$$\epsilon(u) \approx \sqrt{2}(\pi u)^{-1/4}. \tag{4}$$

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A better approximation can be found by trying to match higher order terms, namely a term in $u^{-1/4}$ on the r.h.s. We then use $\epsilon(u) = au^{-1/4} + bu^{-1/2}$, compute the integral (discarding singular terms appearing only for small u, where our approximation is not relevant) and match the first two terms, which lead to the following approximation:

$$\epsilon(u) \approx \sqrt{2}(\pi u)^{-1/4} - \frac{\Gamma(3/4)}{\Gamma(1/4)} u^{-1/2}, \tag{5}$$

or simply $\epsilon(u) \approx (1.0623)u^{-1/4} - (0.3380)u^{-1/2}$.

Early times. In that case we look for solutions $\theta(u)$ of Equation (1) for small u. Linearising the integrand for small u (and therefore for small $\theta(u)$), and integrating by parts, we find that

$$\theta(u) \approx \sqrt{u/\pi} - \frac{1}{\sqrt{\pi}} \int_0^u \frac{\theta(u')}{\sqrt{u-u'}} du'. \tag{6}$$

This is a linear Abel integral equation of the second kind for θ, and can be solved straightforwardly, which yields:

$$\theta(u) \approx (1/2)\left(1 - \exp(u)\mathrm{erfc}(\sqrt{u})\right). \tag{7}$$

2 Fracture energy asymptotics in the slip on plane limit

Throughout this Section we use a normalised time $u = t/t_{\mathrm{w}}^{\mathrm{SP}}$, temperature $\theta = (T - T_0)/(T_{\mathrm{w}} - T_0)$, strength $\tilde{\tau} = \tau/\tau_0$ and slip rate $v = V/V_{\mathrm{w0}}$. The natural characteristic slip scale is therefore $\delta^* = V_{\mathrm{w0}} t_{\mathrm{w}}^{\mathrm{SP}}$, and the characteristic scale for fracture energy is $G^* = \tau_0 \delta^*$. With these notations, the constitutive law (Equation 1 in the main text) becomes:

$$\tilde{\tau} = (1-\theta)^2/v, \tag{8}$$

and the normalised fracture energy $\tilde{G} = G/G^*$ is

$$\tilde{G} = \int_0^{\delta'} (\tilde{\tau}[\delta] - \tilde{\tau}[\delta'])d\delta, \tag{9}$$

where $\delta' = \delta/\delta^*$ is the normalised slip.

At constant slip rate, (9) becomes

$$\tilde{G} = \int_0^u \left((1-\theta(u'))^2 - (1-\theta(u))^2\right)du'. \tag{10}$$

Small slip, constant slip rate. In the small slip limit, slip-on-a-plane regime, an approximate form of the fracture energy can be determined by expanding $\theta(u)$ for small u:

$$\theta(u) \approx \sqrt{u/\pi} - u/2, \tag{11}$$

and then use that approximation in the computation of $\tilde{G}$ in Equation 10. The result is then

$$\tilde{G} \approx \frac{2}{3\sqrt{\pi}} u^{3/2} - \frac{\pi + 1}{2\pi} u^2. \tag{12}$$

Retaining only the leading order term, the dimensional form of (12) reads

$$G \approx G^* \frac{2}{3\sqrt{\pi}} (\delta'/v)^{3/2}, \tag{13}$$

which is the same as Equation (15) in the main text.

Large slip, constant slip rate. In a similar fashion, a large time approximation can be found based on the asymptotic approximation for $\theta(u)$ given in Equation (5). After direct integration of (10), we find:

$$\tilde{G} \approx 2\sqrt{\frac{u}{\pi}} - 6\sqrt{2}\frac{\Gamma(3/4)}{\Gamma(1/4)} \left(\frac{u}{\pi}\right)^{1/4}. \tag{14}$$

Again, retaining only the leading order term yields the following dimensional form for the fracture energy:

$$G \approx 2G^* \sqrt{\frac{\delta'}{\pi v}}, \tag{15}$$

which is the same as Equation (16) in the main text.

Large slip, dynamic crack solution. For a semi-inifinite shear crack propagating at constant speed, we recall that the elastodynamic equilibrium implies that

$$\tau(x) = \frac{\mu^*}{2\pi V_\mathrm{r}} \int_0^\infty \frac{V(s)}{s - x} ds, \tag{16}$$

where x is the position from the rupture tip, V_r is the rupture speed and $\mu*$ is an elastic shear modulus modified according to the rupture speed (*Rice*, 1980). Normalising the position by $x^* = V_\mathrm{r} t_\mathrm{w}^\mathrm{SP}$, Equation 16 is rewritten as

$$\tilde{\tau}(u) = \frac{\mu'}{2\pi} \int_0^\infty \frac{v(u')}{u' - u} du', \tag{17}$$

where

$$\mu' = \frac{\mu^* V_\mathrm{w0}}{\tau_0 V_\mathrm{r}}. \tag{18}$$

Following *Viesca and Garagash* (2015), we look for asymptotic solutions of (17) in the form $v(u) = Bu^\lambda$ and $\tau(u) = -\mu'(B/2)\mathrm{cotan}(\pi\lambda)u^\lambda$ also satisfying (asymptotically for large u) Equation (1) and the constitutive law (8). As observed in the previous Section, at large u, Equation (1) implies that $\theta \approx 1 - \sqrt{2}(\pi u)^{-1/4}$. The constitutive law (8) then implies that:

$$\tilde{\tau} v \approx 2/\sqrt{(\pi u)}. \tag{19}$$

Using $v(u) = Bu^{\lambda}$ and $\tau(u) = -\mu'(B/2)\text{cotan}(\pi\lambda)u^{\lambda}$ in (19) yields:

$$\lambda = -1/4 \qquad \text{and} \qquad B = 2/(\mu'^{1/2}\pi^{1/4}), \tag{20}$$

so that

$$v(u) \approx (2/\sqrt{\mu'})(\pi u)^{-1/4} \quad \text{and} \quad \tilde{\tau}(u) \approx \sqrt{\mu'}(\pi u)^{-1/4}. \tag{21}$$

The (normalised) slip distance is given by

$$\delta' = \int_0^u v(u')du', \tag{22}$$

from which we compute $u \approx [(3/8)\mu'^{1/2}\pi^{1/4}\delta']^{4/3}$. The stress as a function of slip is therefore given by

$$\tau(\delta') \approx \left(\frac{8\mu'}{3\pi}\right)^{1/3} \delta'^{-1/3}, \tag{23}$$

which can be integrated straightforwadly to yield

$$\tilde{G}(\delta') \approx \left(\frac{\mu'}{3\pi}\right)^{1/3} \delta'^{2/3}. \tag{24}$$

The dimensional form of (24) corresponds to Equation (18) of the main text.

3 Comparison of characteristic scales between flash heating and thermal pressurisation

In order to compare the weakening produced by flash heating and thermal pressurisation, it is instructive to compute the ratio $r_{\mathrm{FH/TP}}$ of the characteristic slip weakening distances associated with each process. In the adiabatic regime, the ratio is

$$r^{\mathrm{A}}_{\mathrm{FH/TP}} = \frac{t^{\mathrm{A}}_{\mathrm{w}} V}{\rho c\sqrt{2\pi} w/(f\Lambda)}, \tag{25}$$

where Λ denotes the thermal pressurisation factor (*Rice*, 2006). Assuming that the friction coefficient f relevant for thermal pressurisation is equal to the initial friction coefficient before flash heating operates, Equation 25 simplifies to

$$r_{\mathrm{FH/TP}} = \frac{T_{\mathrm{w}} - T_0}{T_{\max} - T_0}\frac{V}{V_{\mathrm{w0}}}, \quad \text{(adiabatic)} \tag{26}$$

where $T_{\max} = \sigma'_0/\Lambda$ is the maximum temperature rise associated with adiabatic, undrained thermal pressurisation (σ'_0 denotes the initial effective normal stress on the fault). For typical crustal parameter values (*Brantut and Platt*, 2017), $r_{\mathrm{FH/TP}}$ is of the order of 10, which implies that progressive weakening by adiabatic flash heating occurs over much larger slip distances than thermal pressurisation. We note, however, that flash heating has an "instantaneous" effect (occurring over distances of a few tens of microns) that will always occur first if V is significantly greater than V_{w0} (*Brantut and Rice*, 2011). Thermal pressurisation therefore dominates the initial weakening only when $V \lesssim V_{\mathrm{w0}}$. This is the case only for relatively thick gouge layers, since V_{w0} increases proportionally to the number of contacts over which sliding occurs within the gouge.

In the slip-on-a-plane limit, the relevant slip scales for flash heating and thermal pressurisation are $t_{\mathrm{w}}^{\mathrm{SP}}V$ and L^*, respectively, where

$$L^* = \frac{4\alpha^*}{V}\left(\frac{\rho c}{f\Lambda}\right)^2. \tag{27}$$

In the expression for L^*, the diffusivity α^* is a combined hydraulic and thermal diffusivity (e.g. *Garagash*, 2012). In the limit of large hydraulic diffusivity compared to thermal diffusivity, which is most likely the case in nature, α^* is approximately equal to the hydraulic diffusivity itself. The ratio of characteristic slip distances for flash heating and thermal pressurisation is therefore

$$r_{\mathrm{FH/TP}} = \frac{\alpha}{\alpha^*}\left(\frac{T_{\mathrm{w}} - T_0}{T_{\max} - T_0}\frac{V}{V_{\mathrm{w0}}}\frac{f}{f_0}\right)^2, \quad \text{(slip-on-a-plane)} \tag{28}$$

where f is the friction coefficient operating during thermal pressursation, and f_0 is the initial friction coefficient before flash heating occurs. The ratio f/f_0 is likely less than one, and more critically we generally have $\alpha/\alpha^* \ll 1$, so that the ratio $r_{\mathrm{FH/TP}}$ is much less than 1 for realistic crustal fault properties (see *Viesca and Garagash*, 2015; *Brantut and Platt*, 2017).

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